

Relativistic chiral representation of the πN scattering amplitude I: The Goldberger-Treiman relation

J. M. Alarcón¹, J. Martín Camalich² and J. A. Oller¹

¹*Departamento de Física. Universidad de Murcia. E-30071, Murcia. Spain*

²*Department of Physics and Astronomy, University of Sussex, BN1 9QH, Brighton, UK*

November 22, 2011

Abstract

In this work we study the πN scattering process within the Baryon Chiral Perturbation Theory framework in the covariant scheme of Extended-On-Mass-Shell (EOMS). We compare the description obtained in this scheme with the previously obtained using the Infrared Regularization scheme and show that EOMS accomplishes the best convergence, being able to extract from partial wave analyses reliable values of important quantities as the Goldberger-Treiman deviation. In regard to the latter, we solve the long-standing problem concerning to the extraction of the Goldberger-Treiman deviation with covariant ChPT that jeopardized the applicability of ChPT to the πN system. We also show the potential of the unitarization techniques applied to the perturbative calculation in the EOMS scheme, that allow us to increase the range of validity of our description up to ≈ 200 MeV in \sqrt{s} .

1 Introduction

The πN scattering is a process thoroughly studied experimentally and, in fact, we have experimental data since sixties. From the theoretical point of view, it is the basic hadronic process involving baryons and one of the most important test ground for Chiral Perturbation Theory with Baryons (BChPT). The first attempt to study this process using BChPT was performed by Gasser, Sainio and Svarc in their seminal work [1] using a fully covariant approach. In this work, they realized that when one deals with nucleons, a new heavy scale appears in the ChPT formalism that does *not vanishes in the chiral limit* and spoils the standard power counting of ChPT. In order to solve this problem, Jenkins and Manohar invented the Heavy Baryon Chiral Perturbation Theory approach [2] HBChPT, which integrates out the heavy degrees of freedom of the nucleon expanding the Lagrangian in series $1/m_N$, with m_N the nucleon mass. This formalism describes well the low-energy physical region [3, 4, 5, 6] at the cost of losing Lorentz invariance, although it does not converge in the subthreshold region [7, 8]. This means that we cannot check some chiral symmetry predictions for QCD (low energy theorems). In order to solve this problem of convergence, Becher and Leutwyler invented the Infrared Regularization scheme (IR) [9], that recovers the standard power counting of ChPT keeping manifest Lorentz invariance. This improves the convergence with respect to HBChPT and converges in the subthreshold region. However,

as was shown in [8], the one-loop representation is not precise enough to allow an accurate extrapolation of the physical data to the Cheng-Dashen point to extract the value of $\sigma_{\pi N}$. Later works showed that the IR description of the phase shifts are of the same quality as those of HBChPT [10], although a *huge* and *strongly scale dependent* Goldberger-Treiman (GT) relation deviation is found in this scheme [11, 10]. In fact, the scale dependence is one of the main characteristics of IR. Another important limitation of IR comes out when we reach energies that can make the Mandelstam variable $u = 0$ (this energy corresponds to $\sqrt{s} \gtrsim 1.34$ GeV for πN scattering), because in this case the amplitudes develop an unphysical cut that limits the high energy description and, therefore, the applicability of Unitarization methods [10]. This unphysical cut is also responsible for a bad prediction for the magnetic moments when using this scheme [12]. In order to overcome the problems that one encounters in the IR scheme keeping the good analytical properties of a covariant approach, we calculated de πN scattering amplitude in ChPT up to $\mathcal{O}(p^3)$ in the chiral expansion, using the so-called *Extended-On-Mass-Shell* scheme (EOMS) [13]. This scheme recovers the spirit of the full covariant approach performed by [1] but keeping the standard power counting of ChPT through a renormalization of the low energy constants (LECs) that appear in the Lagrangian. So, the EOMS scheme can be considered as a second renormalization in the sense that the first renormalization would be the \overline{MS} renormalization that cancel the infinities that come from the loop diagrams, and the EOMS renormalization is the renormalization that cancel the power counting breaking terms (PCBT) that appear in the covariant approach. The proof that this renormalization can cancel *all* the PCBT comes from the IR formalism because Becher and Leutwyler proved that all the PCBT are contained in what they called the *regular part* of the loop integral, that is analytical in the quark masses and momenta, so that means that can be absorbed in the most general Lagrangian. The advantages of this scheme over the IR one are [14]: 1) we do *not* have to deal with any scale dependence, 2) the contribution of the loop diagrams to the GT deviation is very small ($\approx 0.2\%$) that is of the size of what we would expect from explicit symmetry breaking, 3) our amplitudes are free from unphysical cuts, what means that they have the right analytical properties in the whole energy plane. In this proceeding we will focus on the comparison between both covariant methods: EOMS and IR.

2 Perturbative Calculations

To compare fairly both methods we proceed with EOMS in the same way as we did with IR, so we perform the perturbative study as in [10]. We considered the partial wave analyses (PWAs) of the Karlsruhe group [15] (KA85) and the current solution of the George Washington University group [16] (WI08), assigning the same errors as we did in [10]. The results of the fits are shown in Fig. 1 and Fig. 2

The fitted values of the LECs are given in Table 1. In this table one can compare the EOMS results with those of the IR and HBChPT methods. One can see that the EOMS results for the LECs are compatible with both approaches being able to give a better description than IR (lower $\chi^2_{d.o.f.}$). At this point it is important to stress two things: First, we see in Fig. 1 and Fig. 2 that with the EOMS scheme we do not have the problems that we encountered in [10] when we tried to fit the P_{11} phase shift of WI08 with the IR scheme (dashed line Fig. 2). Second, and more important, with the EOMS scheme we solved the problem of the huge Δ_{GT} of the IR scheme that jeopardized the applicability of ChPT to the πN system. Within the EOMS scheme we can *extract from data* values for Δ_{GT} compatible with the ones reported by the corresponding PWAs. Although the values presented in Table 1 for the EOMS result may be considered not very accurate and quite large for KA85-EOMS, it is important to stress that these results can be considerably improved once we include explicitly the $\Delta(1232)$ in our EOMS calculation [17]. In this case we obtain very accurate predictions for Δ_{GT} that are perfectly compatible with their corresponding PWAs [17]. For a deeper understanding of what is happening in the covariant

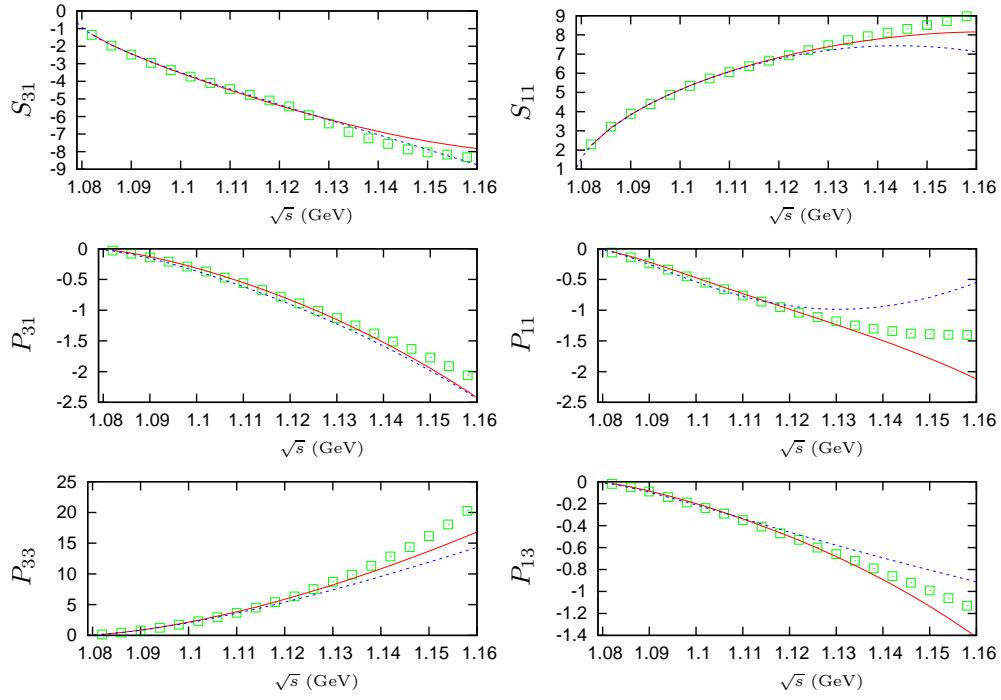


Figure 1: Fits to KA85 [15]. Solid line corresponds to the EOMS result and the dashed to IR. Both fits are performed up to $\sqrt{s}_{max} = 1.13$ GeV

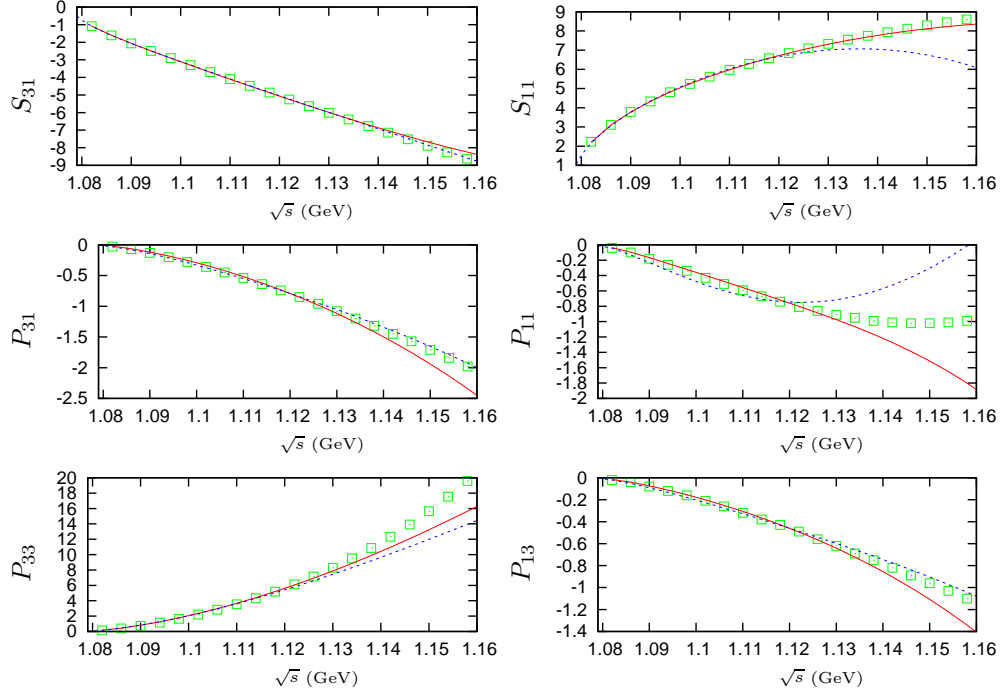


Figure 2: Fits to WI08 [16]. Solid line corresponds to the EOMS result and the dashed to IR. Both fits are performed up to $\sqrt{s}_{max} = 1.13$ GeV

LEC	KA85-EOMS $\mathcal{O}(p^3)$	WI08-EOMS $\mathcal{O}(p^3)$	KA85-IR $\mathcal{O}(p^3)$ [10]	WI08-IR $\mathcal{O}(p^3)$ [10]	HBChPT $\mathcal{O}(p^3)$ [3]
c_1	-1.26 ± 0.07	-1.50 ± 0.06	-0.71 ± 0.49	-0.27 ± 0.51	$(-1.71, -1.07)$
c_2	4.08 ± 0.09	3.74 ± 0.09	4.32 ± 0.27	4.28 ± 0.27	$(3.0, 3.5)$
c_3	-6.74 ± 0.08	-6.63 ± 0.08	-6.53 ± 0.33	-6.76 ± 0.27	$(-6.3, -5.8)$
c_4	3.74 ± 0.05	3.68 ± 0.05	3.87 ± 0.15	4.08 ± 0.13	$(3.4, 3.6)$
$d_1 + d_2$	3.25 ± 0.55	3.67 ± 0.54	2.48 ± 0.59	2.53 ± 0.60	$(3.2, 4.1)$
d_3	-2.72 ± 0.51	-2.63 ± 0.51	-2.68 ± 1.02	-3.65 ± 1.01	$(-4.3, -2.6)$
d_5	0.50 ± 0.13	-0.07 ± 0.13	2.69 ± 2.20	5.38 ± 2.40	$(-1.1, 0.4)$
$d_{14} - d_{15}$	-6.10 ± 1.08	-6.80 ± 1.07	-1.71 ± 0.73	-1.17 ± 1.00	$(-5.1, -4.3)$
d_{18}	-2.96 ± 1.44	-0.50 ± 1.43	-0.26 ± 0.40	-0.86 ± 0.43	$(-1.6, -0.5)$
$\chi^2_{d.o.f.}$	0.35	0.22	$\lesssim 1$	$\lesssim 1$	-
Δ_{GT}	$9 \pm 4\%$	$2 \pm 4\%$	$(20 - 30\%)$	$(20 - 30\%)$	(input)

Table 1: Comparison between LECs and the resulting Δ_{GT} in the different approaches of BChPT.

methods, we will explain briefly the method used for the extraction of the GT deviation. As we did in [10], the method consist in taking the limit :

$$\lim_{s \rightarrow m_N^2} \frac{T^{\mathcal{O}(p^3)}}{T^{\mathcal{O}(p)}} = \left(\frac{g_{\pi N}}{g_A m_N / f_\pi} \right)^2 = (1 + \Delta_{GT})^2$$

Where $T^{\mathcal{O}(p^3)}$ and $T^{\mathcal{O}(p)}$ mean the full amplitude calculated up to $\mathcal{O}(p^3)$ and $\mathcal{O}(p)$ respectively. In ChPT Δ_{GT} is directly related to the LEC d_{18} plus a higher order contribution due to the loops (Δ_{loops}):

$$\Delta_{GT} = -\frac{2M_\pi^2 d_{18}}{g_A} + \Delta_{loops}$$

From fits to PWAs one obtains a natural value for d_{18} from both covariant schemes (EOMS and IR), but when one calculates explicitly the value of Δ_{loops} it turns out that the IR scheme gives huge values that depend strongly on the renormalization scale (20–30%) [10] while EOMS gives a *scale-independent value that is of the size that we would expect from explicit chiral symmetry breaking* ($\approx 0.2\%$), solving the long standing problem that covariant BChPT had with this observable. This means that the huge Δ_{GT} is due to the IR prescription, not to a problem of BChPT.

3 Unitarized Calculations

Another limitation that we encountered in the IR scheme concerns to the applicability of Unitarization techniques to the perturbative calculation. In IR we found that Unitarization techniques are limited up to $\sqrt{s} \approx 1.25$ GeV due to the unphysical cut that this scheme introduces [10]. So, it is interesting to study if a covariant calculation without this unphysical cut could improve the description of the phase shifts. With this aim we implement unitarity to the EOMS-BChPT πN amplitude and take care of the analyticity properties associated with the right-hand cut writing our unitarized amplitude $T_{IJ\ell}$ by means of an interaction kernel $\mathcal{T}_{IJ\ell}$ and the unitary pion-nucleon loop function $g(s)$:

$$T_{IJ\ell} = \frac{1}{\mathcal{T}_{IJ\ell}^{-1} + g(s)}$$

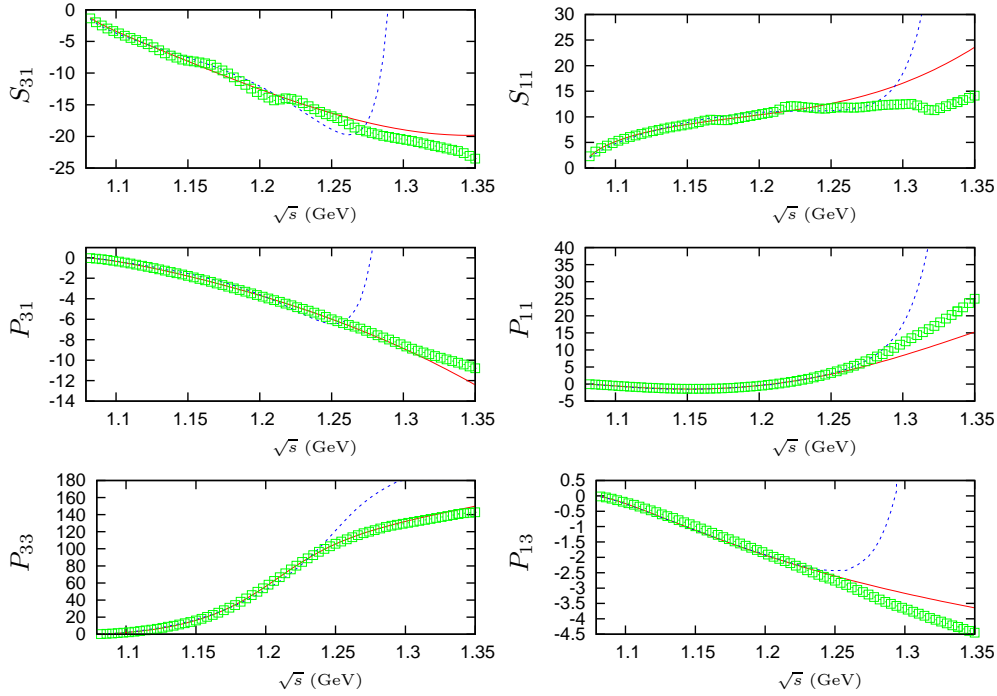


Figure 3: Unitarized fits to KA85. Solid line: EOMS. Dashed line: IR [10]

Where $T_{IJ\ell}$ is the amplitude with definite isospin I , total angular momentum J and orbital angular momentum ℓ . Written in this form, $T_{IJ\ell}$ satisfies unitarity *exactly* and the interaction kernel $\mathcal{T}_{IJ\ell}$ can be obtained by matching order by order with the perturbative result [18]. On the other hand, the subtraction constant a_1 contained in the unitary pion-nucleon loop function $g(s)$ is fixed by requiring that g vanishes in the nucleon pole $g(m_N^2) = 0$ so in this point we recover the perturbative calculation and keep the P_{11} nucleon pole in its right position. On the other hand, in order to take into account the contribution of the $\Delta(1232)$ we introduce, in the P_{33} partial wave, a Castillejo-Dalitz-Dyson pole (CDD) [19] that is a pole that conserves the discontinuities of the partial wave amplitude across the cuts. For this partial wave, the unitarized amplitude reads: $T_{\frac{3}{2}\frac{3}{2}1} = \left(\mathcal{T}_{\frac{3}{2}\frac{3}{2}1}^{-1} + \frac{\gamma}{s-s_P} + g(s) \right)^{-1}$, where the CDD corresponds to $\frac{\gamma}{s-s_P}$, and gives rise to a zero at s_P in $T_{\frac{3}{2}\frac{3}{2}1}$.

In both Fig. 3 and Fig. 4 one can see how the kernel calculated in the EOMS scheme provides a good description of the phase shifts up to $\sqrt{s} \approx 1.35$ GeV while IR can only reproduce them up to $\sqrt{s} \approx 1.25$ GeV due to the unphysical cut that this scheme introduces. This corresponds to an increase of ≈ 100 MeV in \sqrt{s} compared to IR. On the other hand the CDD is able to reproduce perfectly the raise of the P_{33} phase shift due to the $\Delta(1232)$, and if one compares the description of the unitarized and the perturbative amplitudes, one finds a drastic increase in the energy region of the data (≈ 200 MeV in \sqrt{s}).

4 Summary and Conclusions

In summary, the πN scattering is a fundamental process that provides the basic test for ChPT with baryons. There have been many attempts to describe this process in BChPT but every approach has had their own problems: *lack of convergence*, *unphysical cuts*, *unphysically large GT deviation*, etc. These problems questioned the applicability of ChPT to the πN system. In this work we showed that BChPT in the EOMS scheme solves these issues providing a chiral representation that converges, and giving rise to a GT violation in good agreement with phenomenology. This and other important quantities

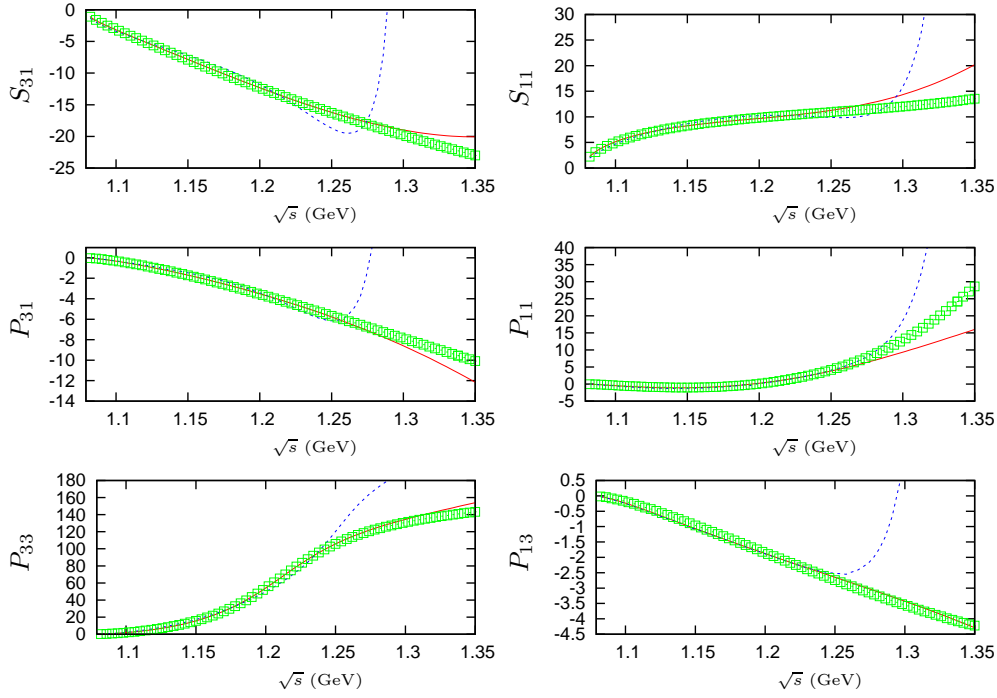


Figure 4: Unitarized fits to WI08. Solid line: EOMS. Dashed line: IR [10]

such as the $\sigma_{\pi N}$ can be extracted from PWAs accurately and reliably once we introduce explicitly the contribution of the $\Delta(1232)$ in our calculations, as we showed in [17], and that part will be explained in [21]. It is also very interesting to show the potential of the Unitarization techniques applied to a kernel with good analytical properties. In this work we show that with this unitarization method we could increase the range of our description up to $\sqrt{s} \approx 1.35$ GeV, that means an improvement of ≈ 200 MeV in \sqrt{s} with respect to the perturbative calculation. Compared with IR, the unitarized EOMS amplitudes achieve a good description up to energies ≈ 100 MeV higher in \sqrt{s} .

References

- [1] J. Gasser, M. E. Sainio, A. Svarc, Nucl. Phys. **B307** (1988) 779.
- [2] E. E. Jenkins, A. V. Manohar, Phys. Lett. **B255** (1991) 558-562.
- [3] N. Fettes, U. -G. Meissner, S. Steininger, Nucl. Phys. **A640** (1998) 199-234. [hep-ph/9803266].
- [4] N. Fettes, U. -G. Meissner, Nucl. Phys. **A676** (2000) 311. [hep-ph/0002162].
- [5] N. Fettes, U. G. Meissner, Nucl. Phys. **A679** (2001) 629-670. [hep-ph/0006299].
- [6] P. Buettiker, U. -G. Meissner, Nucl. Phys. **A668** (2000) 97-112. [hep-ph/9908247].
- [7] V. Bernard, N. Kaiser, U. -G. Meissner, Phys. Rev. Lett. **74** (1995) 3752-3755. [hep-ph/9412282].
- [8] T. Becher, H. Leutwyler, JHEP **0106** (2001) 017. [hep-ph/0103263].
- [9] T. Becher, H. Leutwyler, Eur. Phys. J. **C9** (1999) 643-671. [hep-ph/9901384].
- [10] J. M. Alarcon, J. M. Camalich, J. A. Oller, L. Alvarez-Ruso, Phys. Rev. **C83** (2011) 055205. [arXiv:1102.1537 [nucl-th]].

- [11] K. Torikoshi, P. J. Ellis, Phys. Rev. **C67** (2003) 015208. [nucl-th/0208049].
- [12] L. S. Geng, J. Martin Camalich, L. Alvarez-Ruso, M. J. Vicente Vacas, Phys. Rev. Lett. **101** (2008) 222002. [arXiv:0805.1419 [hep-ph]].
- [13] T. Fuchs, J. Gegelia, G. Japaridze, S. Scherer, Phys. Rev. **D68** (2003) 056005. [hep-ph/0302117].
- [14] J. M. Alarcón, J. Martín Camalich and J. A. Oller, In preparation.
- [15] R. Koch, Nucl. Phys. **A448** (1986) 707.; R. Koch, E. Pietarinen, Nucl. Phys. **A336**, 331-346 (1980).
- [16] Computer code SAID, online program at <http://gwdac.phys.gwu.edu/> , solution WI08. R. A. Arndt et al., Phys. Rev. **C74** (2006) 045205. solution SM01.
- [17] J. M. Alarcon, J. M. Camalich, J. A. Oller, [arXiv:1110.3797 [hep-ph]].
- [18] J. A. Oller, U. G. Meissner, Phys. Lett. **B500** (2001) 263-272. [hep-ph/0011146].
- [19] L. Castillejo, R. H. Dalitz, F. J. Dyson, Phys. Rev. **101** (1956) 453-458.
- [20] J. A. Oller, E. Oset, Phys. Rev. **D60** (1999) 074023. [hep-ph/9809337].
- [21] “*The chiral representation of the πN scattering amplitude and the pion-nucleon sigma term*”, talk by Dr. Jorge Martín Camalich.